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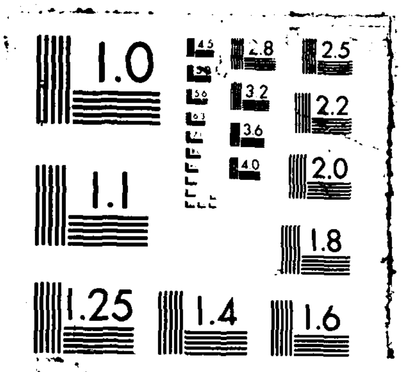
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ABSTRACT

Let ϕ be a parameter for which there is no unbiased estimator. This note shows that for an arbitrary sequence of estimators $T^{(k)}$, if the biases of $T^{(k)}$ tend to 0 then their variances must tend to ∞.

1. INTRODUCTION.

Let $X = (X_1, \ldots, X_n)$ have distribution P_θ, where the unknown parameter varies in Θ. Suppose that we need to estimate a real valued function $\phi(\theta)$ of the parameter. Let $\hat{\phi} = \hat{\phi}(X)$ be a biased estimator of ϕ. There exist several procedures for reducing the bias of $\hat{\phi}$: jackknifing, bootstrapping (see Efron (1982)), and other procedures based on expansions of $E_\theta(\hat{\phi})$(see Cox and Hinkley (1974, Section 8.4)). These procedures may not eliminate the bias completely, and one often hears the following suggestion. Let $\hat{\phi}^{(1)}$ be obtained from $\hat{\phi}$ by one of these bias-reduction procedures. If $\hat{\phi}^{(1)}$ is still biased, repeat the bias-reduction procedure and obtain $\hat{\phi}^{(2)}, \hat{\phi}^{(3)}$ etc. until a desired amount of reduction in bias is obtained or the bias is removed completely. Such "higher-order bias corrections" are described for instance in the review paper of Miller (1974) in connection with the jackknife. There are examples where no unbiased estimator of ϕ exists but there exists a sequence of estimators $\hat{\phi}, \hat{\phi}^{(1)}, \hat{\phi}^{(2)}, \ldots$ whose biases converge to zero (see Section 2).

The purpose of this note is to show (Theorem 1) that when no unbiased estimator of ϕ exists, then reducing the bias to zero necessarily forces the variance of the estimators to tend to ∞. This theorem therefore gives qualitative support to the widely held view that bias reduction is by itself not a desirable property, but becomes desirable only if it can be demonstrated that it is accompanied by a reduction in mean squared error.

2. MAIN RESULT AND REMARKS.

Let $(\mathcal{X}, \mathcal{S})$ be a measurable space and $(P_\theta, \theta \in \Theta)$ be a family of probability measures on $(\mathcal{X}, \mathcal{S})$. Let ϕ be a real valued function defined on Θ. The bias of an estimator $T = T(X)$ is defined by $\beta_T(\theta) = E_\theta(T(X)) - \phi(\theta)$, assuming that $E_\theta(T(X))$ exists.

THEOREM 1. Suppose that

(A1) $P_{\theta_1} \ll P_{\theta_2}$ for all θ_1, θ_2 in Θ,

(A2) $\int (\frac{dP_{\theta_1}}{dP_{\theta_2}})^2 dP_{\theta_2} < \infty$ for all θ_1, θ_2 in Θ,

and that $\{T_k\}_{k=1}^{\infty}$ is a sequence of estimators for which

$$\beta_{T_k}(\theta) \to 0 \text{ for all } \theta \text{ in } \Theta. \tag{1}$$

If there does not exist an unbiased estimator of ϕ then

$$\mathrm{Var}_\theta(T_k) \to \infty \quad \text{as } k \to \infty, \quad \text{for all } \theta \in \Theta. \tag{2}$$

Proof: Suppose that (2) is not true. Then there exists a θ_0 in Θ and a subsequence $\{k^*\}$ of $\{k\}$ such that $\mathrm{Var}_{\theta_0}(T_{k^*})$ is bounded. Now, consider the usual Hilbert space $H_{\theta_0} = L^2(\mathcal{X}, S, P_{\theta_0})$ of all functions that are square-integrable with respect to P_{θ_0}. Notice that $\{T_{k^*}\}$ is a norm-bounded set in H_{θ_0}. From the sequential weak-compactness of norm-bounded sets, there exists a T in H_{θ_0} and a subsequence $\{k^{**}\}$ of $\{k^*\}$ such that $T_{k^{**}} \to T$ weakly in H_{θ_0} along the subsequence $\{k^{**}\}$, i.e.

$$\int T_{k^{**}} f dP_{\theta_0} \to \int T f dP_{\theta_0} \text{ for every function } f \text{ in } H_{\theta_0}.$$

In particular, setting $f = dP_\theta / dP_{\theta_0}$, we get

$$E_\theta(T_{k^{**}}) \to E_\theta(T),$$

along the subsequence $\{k^{**}\}$, for all θ in Θ. From (1), it now follows that $E_\theta(T) = \phi(\theta)$, that is T is unbiased for ϕ, which contradicts one of our assumptions. Hence (2) holds and the proof is complete. ∎

There are many examples of situations to which this theorem applies. One class can be obtained from the idea of the following example. Consider the family of Poisson distributions with parameter λ with $\lambda > 0$. It is well known that there exists no unbiased estimator of $1/\lambda$, and that all polynomials in λ are unbiasedly estimable. From (a slight modification of) the Stone-Weirstrass theorem, there exists a sequence of polynomials in λ which converge to $1/\lambda$ for each λ. Thus there exists a sequence of estimators which are unbiased for these polynomials in λ, and whose biases in estimating $1/\lambda$ converge to zero. A simple calculation shows that $\int \left(\frac{dP_{\lambda_1}}{dP_{\lambda_2}}\right)^2 dP_{\lambda_1} = \exp(\lambda_2 - 2\lambda_1 + \lambda_1^2/\lambda_2)$. Thus Theorem 1 applies to this case and the variances of these estimators must tend to ∞.

It may appear that Theorem 1 does not apply to estimates based on the jackknife, since the "delete-one" jackknife can be formed only a finite number of times. However, a situation with an infinite sequence of estimators based on the jackknife arises in the following example, based on an idea of Gaver and Hoel (1970). Suppose that the data consists of a Poisson process $\{N(t); t \in [0,1]\}$ with rate λ. In connection with the biased maximum likelihood estimator $\hat{\phi} = e^{-\lambda N(1)}$ of $e^{-\lambda}$, Gaver and Hoel suggest splitting the interval $[0,1]$ into n nonoverlapping intervals each of length $1/n$, and letting N_i be the number of events in the ith interval. These are independent and identically distributed and one can therefore form the delete-one jackknife as usual. This yields, for each n, an estimate $\hat{\phi}_{(n)}$ and they show that as $n \to \infty$ $\hat{\phi}_{(n)}$ converges to an estimate $\hat{\phi}^{(1)}$

which depends on the Poisson process only through the sufficient statistic $N(1)$. This procedure can be repeated indefinitely in principle, giving a sequence of estimators $\{\hat{\phi}^{(k)}\}_{k=1}^{\infty}$.

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